PROBLEMS SECTION 4
MATH 10A, Summer Session E, UC Riverside 2018

Section 4.2

Problem 103. *Find the arc length of* $\vec{c}(t) = (2t)\vec{i} + (\ln t)\vec{j} + (t^2)\vec{k}$ *for* $1 \leq t \leq 2$.

Problem 104. *Find the arc length of* $\vec{c}(t) = (\sin 3t)\vec{i} + (\cos 3t)\vec{j} + 2t^{\frac{3}{2}}\ \vec{k}$ *for* $1 \leq t \leq 2$.

Section 4.3

Problem 105. *Show that the path* $\vec{c}(t) = (2\cos t, 2\sin t)$ *is a flow line of the vector field*
$$\vec{F}(x, y) = -y\vec{i} + x\vec{j}.$$

Problem 106. *Show that the path* $\vec{c}(t) = (t^2, 2t - 1, \sqrt{t}),\ t > 0$ *is a flow line of the vector field*
$$\vec{F}(x, y, z) = (y + 1, 2, \frac{1}{2z}).$$

Section 4.4

Problem 107. *Given a function* $f : \mathbb{R}^3 \to \mathbb{R}$, *the Laplacian of* f, *denoted by* Δf, *is a very important differential operator. It is defined to be* $\Delta f := \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$.

a) *Show that* $\Delta f = \mathrm{tr}\ \mathrm{Hess} f$. *Remember that the trace (*tr) of a matrix is the sum of the terms in its diagonal positions.

b) Show that $\Delta f = \nabla \cdot (\nabla f)$.

Problem 108. *Let* $f(x, y, z) = x^2 + y^2 + z^2$. *Compute* ∇f *and then compute* $\nabla \times (\nabla f)$.

Problem 109. *Let* $\vec{v}(x, y, z) = (xy, yz, zx)$. *Compute* $\nabla \times \vec{v}$ *and then compute* $\nabla \cdot (\nabla \times \vec{v})$.

Problem 110. *Calculate the divergence and curl of the vector fields* $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$.

Problem 111. *Let* $\vec{F}(x, y, z) = (x^2, x^2 y, z + zx)$. *Compute* $\nabla \cdot (\nabla \times \vec{F})$. *Show that, given any vector field* $\vec{F}(x, y, z) = (f_1, f_2, f_3)$, *the identity*
$$\nabla \cdot (\nabla \times v) = 0$$
is always satisfied. This identity is important in the study of electromagnetism.

Problem 112. *Show that if* $f : \mathbb{R}^3 \to \mathbb{R}$ *then*
$$\nabla \times (\nabla f) = 0$$
This is another identity important in the study of electromagnetism. In fact, something stronger is true: if a vector field $\vec{v} : \mathbb{R}^3 \to \mathbb{R}^3$ *satisfies* $\nabla \times \vec{v} = 0$, *then there exists some function* $f : \mathbb{R}^3 \to \mathbb{R}$ *such that* $\vec{v} = \nabla f$.

Problem 113. *Consider the vector field* $\vec{v}(x, y, z) = (y, x, 1)$. *In the view of the problem above, is there any function* $f : \mathbb{R}^3 \to \mathbb{R}$ *such that* $\vec{v} = \nabla f$*?*