PROBLEMS SECTION 3
MATH 10A, Summer Session E, UC Riverside 2018

Section 3.2

Problem 88. *Let* $f(x,y) = 4e^{x-2y}$. *Recall that* $e^0 = 1$.

(a) *Find the equation for the tangent plane to the graph of* f *at the point* $(4,2)$.
(b) *Find a linear approximation to the expression* $4e^{4.01-2\cdot(1.99)}$.

Problem 89. *Approximately compute* $11^3 \cdot 9$ *using the first-order Taylor series for the function* $f(x,y) = x^3 y$ *at the point* $(x,y) = (10,10)$.

Problem 90. *The second-order Taylor series of the function* $f(x,y) = x^3y^3$ *at the point* $(x,y) = (1,1)$ *can be written as:*

$$f(1+h, 1+k) \sim 1 + ah + bh + Ah^2 + 2Bhk + Ck^2$$

Compute the numbers a, b, A, B *and* C. *Use these numbers and the second-order Taylor series to approximately calculate* $f(1.1, 1.1)$.

Problem 91. *Approximately compute* $1.1^2 \cdot 2.1^2$ *using the first-order Taylor series for the function* $f(x,y) = x^2y^2$ *at the point* $(x,y) = (1,2)$. *How does this result change if you use the second-order Taylor series instead?*

Problem 92. *Find the first order Taylor polynomial of the function* $f(x,y,z) = e^x \cos(yz)$ *near the point* $(0,1,\pi)$.

Problem 93. *Find the second order Taylor polynomial of the function* $f(x,y) = x^2 + 2xy + y^2$ *near the point* $(1,1)$. *What is the remainder* R_2 *of the order* 2 *approximation of* f *in this case?*

Section 3.3

Problem 94. *Find all the critical points of the following functions. For each one, figure out if they are a local maximum, a local minimum, or a saddle point.*

a) $f(x,y) = x^4 - x^2 + y^3 - y$
b) $f(x,y) = x^3 - 3x + y^3 - 3y$
c) $f(x,y) = e^{1-x^2-y^2}$
d) $f(x,y) = \ln(1 + x^2 + y^2)$
e) $f(x,y) = y^2 - x^3$
f) $f(x,y) = x^2 + xy^2 + y^4$
g) $f(x,y) = (x-1)^2 + (x-y)^2$
h) $f(x,y) = x^4 + y^4 - 2y^2 + 4xy - 2x^2$

Problem 95. *Find all the critical points of the following functions. For each one, figure out if they are a local maximum, a local minimum, or a saddle point.*

a) $f(x,y,z) = 2x^2 + 3y^3 - \frac{3}{2}y^2 + z^2 + 20$
b) $g(x,y,z) = y^3 - 3x^2 - 12xy - z^2$

Problem 96. *Consider the function* $f(x,y) = (y - 3x^2)(y - x^2)$.

a) *Show that the origin is the only critical point, and that the usual method for classifying it using the Hessian matrix doesn't work.*
b) *Approach the origin along a line* $l(t) = (at, bt)$ *for* $a, b \in \mathbb{R}$ *not both* 0. *Show that* $\phi(t) := f(l(t))$ *has a local minimum at* $t = 0$.

c) *Approach the origin along the curve* $c(t) = (t, 2t^2)$*; show that* $\psi(t) := f(c(t))$ *has a local maximum at* $t = 0$.
d) *In view of the above, what kind of critical point is the origin?*

> **Remark:** *Notice that* ϕ *and* ψ *are functions of a single variable (like the ones in MATH 9A), so you can use elementary calculus techniques to determine if a point is a local maximum or minimum.*

> **Hint:** *for simplicity, in part b) you can choose* $a = 0$ *and* $b = 1$.

Problem 97. *Consider the function* $f(x, y) = (x^2 + y^2 + 2x)x^3$.
a) *Draw the level set of* f *corresponding to the value* $C = 0$.
b) *This level curve divides the plane* $\mathbb{R}^2$ *in several regions. Find the sign of* f *in each of these regions.*
c) *Use this information to determine if* $(0, 0)$ *is a local maximum, a local minimum, or a saddle point.*

Section 3.4

Problem 98. *Find the minimum distance between the curves* $x + y = 4$ *and* $x^2 + 4y^2 = 4$.

Hint: *use the Lagrange multipliers method on the distance from a point to the line* $x + y = 4$*, restricted to the ellipse* $x^2 + 4y^2 = 4$.

Problem 99. *Find three positive numbers whose product is equal to* 64 *and whose sum is minimal.*

Problem 100. *Write the number* 270 *as a sum of three numbers so that the sum of the products taken two at a time is a maximum.*

Problem 101. *Find the absolute maximum and minimum for*

$$f(x, y, z) = x^2 + y^2 + z^2 - x + y$$

when $x^2 + y^2 + z^2 \leq 1$.

Problem 102. *Consider the plane* $x + 2y - z = -6$ *and the point* $P = (1, 2, -1)$*. In this problem, we will find the point* Q *in the plane that's closest to* P *using* 3 *different methods.*
(1) *Geometric construction: find the line perpendicular to the plane that goes through* P*. Then intersect that line with the plane to find* Q.
(2) *Lagrange multipliers: use the distance function* $d(x, y, z)$ *from* P *to a point* (x, y, z) *in* $\mathbb{R}^3$*, and find the absolute minimum of that function restricted to the plane* $x + 2y - z = -6$.
(3) *Local minimum: write a parametric equation for the plane* $\pi(t, s)$*. Then consider the distance function* $d(t, s)$ *from a point in the plane* $\pi(t, s)$ *and* P*. Look for the local minimum of* $d(t, s)$ *to find* Q.