PROBLEMS SECTION 1
MATH 10A, Summer Session E, UC Riverside 2018

Single Variable Calculus Review

Problem 1. *Find the derivatives of the following functions.*

(1) $f(x) = \sqrt{x^2 + 9}$
(2) $g(x) = e^{x^2+4x}$
(3) $f(x) = \cos(x\pi) + (\ln 10)x$
(4) $f(x) = x^2 \sin(x)$
(5) $y(x) = \frac{\ln(x)}{\sin(x)}$
(6) $z(x) = \ln(x + 2018)$
(7) $z(t) = 3\sin(\pi t)$
(8) $y(z) = \frac{e^{z+4}}{z^2}$
(9) $f(t) = \cos(\ln(t) + 4)$

Problem 2. *Find the following limits:*

(1) $\lim_{x\to 1}(1 - x)\cos(x)$
(2) $\lim_{x\to 0} \frac{e^x-1}{x}$
(3) $\lim_{x\to\infty} \frac{3x^2-2\sin(x)}{x^2+1}$

Problem 3. *Find the value of a so that the following function is continuous at 0:*

$$f(x) = \begin{cases} x^2 + a & \text{if } x \leq 0 \\ -2x + 1 & \text{if } x > 0 \end{cases}$$

Problem 4. *Find the local minima and maxima of the function $f(x) = x^4 - 2x^2 + 12$.*

Problem 5. *Find the Taylor polynomial of degree 2 of $f(x) = \cos(x)$ near $x = 0$.*

Problem 6. *Use the Taylor polynomial of degree 3 of $f(x) = e^x$ near $x = 0$ to approximate the value of e (note that $f(1) = e$).*

section 1.1

Problem 7. *Sketch the vectors $\mathbf{v} = (2, 1)$ and $\mathbf{w} = (1, 2)$. On your sketch, draw in $-\mathbf{v}$, $\mathbf{v}+\mathbf{w}$ and $\mathbf{v} - \mathbf{w}$.*

Problem 8. *Consider the points $A = (2, 0, 1)$ and $B = (-4, 2, -3)$. Show, using vectors, that the midpoint of the segment joining A and B is $M = (-1, 1, -1)$ (do not use the midpoint formula). What are the coordinates of the point in the segment AB whose distance to B is twice its distance to A?*

Problem 9. *Write a parametric equation describing the line passing through points $(0, 0, 1)$ and $(2, 3, 0)$.*

Problem 10. *Write a parametric equation of the line $l_1(t)$ that is parallel to the line found in the previous problem and goes through the origin $(0, 0, 0)$. Does this line intersect the line $l_2(s) = (3, 5, 3) + s(1, 1, -1)$? If so, find the point of intersection P, and the values of the parameters t and s so that $l_1(t) = P$ and $l_2(s) = P$.*

Problem 11. *Find the point of intersection of the line $x = 4 + 2t, y = 7 + 8t, z = -3 + t$, that is, $l(t) = (4 + 2t, 7 + 8t, -3 + t)$, with the xy coordinate plane.*

Problem 12. *Find a parametric equation of the line in* $\mathbb{R}^2$ *given by the equation* $y - 1 = 2(x+1)$. *Conversely, given the parametric equation of the line* $l(t) = (1+t, 1-t)$, *find an equation for that line of the form* $y - y_0 = m(x - x_0)$, *for some numbers* $x_0, y_0, m \in \mathbb{R}$.

Problem 13. *Find the parametric equation of the plane that contains the lines* $l_1(t) = (1,0,0) + t(2,1,0)$ *and* $l_2(s) = (1,0,0) + s(0,1,1)$. *Does this plane contain the point* $(3,0,-1)$*? And the point* $(0,0,0)$*?*

Problem 14. *Find the parametric equation of the plane* $\pi(t,s)$ *containing the parallel lines* $l_1(t) = (1,0,1) + t(1,1,-1)$ *and* $l_2(s) = (2+s, s, -s)$. *Does this plane contain the point* $(2,2,1)$*? What about the point* $(3,1,-1)$*? Does it intersect the line* $l_3(r) = (0,0,0) + r(1,0,0)$ *(i.e., the* $x-$*axis)? If so, find the point of intersection* P, *and the values of the parameters* $t, s, r \in \mathbb{R}$ *such that* $\pi(t,s) = P$ *and* $l_3(r) = P$.

SECTION 1.2

Problem 15. *Find the angle between* $7\mathbf{j} + 17\mathbf{k}$ *and* $-3\mathbf{i} - \mathbf{j}$.

Problem 16. *Consider the line* $l(t) = (1+t, 2-t, 4)$. *What is the point of intersection of this line with the plane* $x + 2y = 0$*?*

Problem 17. *Let* $\mathbf{v} = (a,b,c)$ *be a vector in* $\mathbb{R}^3$. *Remember that the length of this vector is* $\|\mathbf{v}\| = \sqrt{a^2+b^2+c^2}$.

(1) *Compute the length of the vector* $\mathbf{w} := \frac{\mathbf{v}}{\|\mathbf{v}\|} = (\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}})$.
(2) *If* $\alpha > 0$, *compute the length of* $\mathbf{u} := \alpha \frac{\mathbf{v}}{\|\mathbf{v}\|}$. *What is the angle between* $\mathbf{u}$ *and* $\mathbf{v}$*?*
(3) *What happens if* $\alpha < 0$*? What is the length of* $\mathbf{u}$*? What is the angle between* $\mathbf{u}$ *and* $\mathbf{v}$*?*
(4) *Find a vector of length* 3 *in the direction of* $\mathbf{v} = (3,4,0)$.

Problem 18. *Find a unit vector pointing in the opposite direction from* $(4,0,3)$.

Problem 19. *What is the angle between the vectors* $\mathbf{v} = (2,0,1)$ *and* $\mathbf{w} = (1,-1,1)$*?*

Problem 20. *Suppose* $\mathbf{v}$ *has length* 2, $\mathbf{w}$ *has length* 3, *and the angle between these vectors is* $\pi/4$. *What is* $\mathbf{v} \cdot \mathbf{w}$*?*

Problem 21. *Consider a square. Knowing that two of its adjacent vertices are* $A = (1,-1,3)$ *and* $B = (1,2,-1)$, *what is the area of the square?*

Problem 22.
(1) *Show that, in dimension* 2, *if* $\mathbf{v} = (v_1, v_2)$, *then the vector* $\mathbf{u} = (v_2, -v_1)$ *is perpendicular to* $\mathbf{v}$. *How many vectors are there that are perpendicular to* $\mathbf{v}$*?*
(2) *Show that* $\mathbf{u}$ *has the same length as* $\mathbf{v}$. *How many vectors are there that are perpendicular to* $\mathbf{v}$ *and have the same length as* $\mathbf{v}$*?*
(3) *Would the answer to the last question change in dimension* 3*?*
(4) *Find, without doing any calculations, a vector perpendicular to* $\mathbf{v} = (-1,2)$.

Problem 23. *Find* b *so that* $(3,b,2)$ *is orthogonal to* $\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$.

Problem 24. *Consider the segment joining the points* $A = (2,1)$ *and* $B = (-1,-1)$. *Find the parametric equation of the line that bisects this segment perpendicularly (i.e. the line that is perpendicular to the segment and goes through its midpoint). Fix any point in this line, call it* P *(in general, it depends on* t*, but here you can choose a value for* t *if you want to); what is the distance from* P *to* A*? What is the distance from* P *to* B*? What can you conclude from this?*

Problem 25. *Let* $l(t) = (2+t, 1-t, -3+t)$ *be a line, and consider the point* $P = (2,0,1)$.
(1) *Is* P *a point in the line* l*?*
(2) *What is the distance between a given point in the line* $l(t)$ *and* P*?*

(3) *Using calculus, find the closest point to P in the line l. Let's call it Q.*
(4) *What is the angle between the segment PQ and the line l?*
(5) *Because of Pythagoras theorem, you could have answered part d) without any calculations. Why?*

Problem 26. *Find the projection of* $\mathbf{u} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$ *onto* $\mathbf{v} = 6\mathbf{i} + \mathbf{j} - 9\mathbf{k}$.

SECTION 1.3

Problem 27. *Given the vectors* $\mathbf{u} = (1, -9, 1)$ *and* $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$*, compute:*
(1) $\mathbf{u} + \mathbf{v}$
(2) $\mathbf{u} \cdot \mathbf{v}$
(3) $\|\mathbf{u}\|$
(4) $\mathbf{u} \times \mathbf{v}$
(5) *Area of the parallelogram with sides* $\mathbf{u}$ *and* $\mathbf{v}$.

Problem 28. *Find a vector in* $\mathbb{R}^3$ *that is orthogonal to both* $(1, 2, 3)$ *and* $(3, 2, 1)$.

Problem 29. *Give an equation for the plane containing the point* $p = (1, 2, 0)$ *and orthogonal to the vector* $\mathbf{v} = (2, 1, 3)$.

Problem 30. *Give an equation for the plane containing the point* $p = (6, 7, 1)$ *and orthogonal to the line* $l(t) = (1, 0, -1) + t(4, 1, 0)$.

Problem 31. *Give an equation for the plane containing the point* $p = (1, 0, 0)$ *and parallel to the vectors* $\mathbf{v} = 2\mathbf{i} + \mathbf{j}$ *and* $\mathbf{u} = (1, 0, 1)$.

Problem 32. *Give an equation for the plane containing the points* $A = (1, 3, -5)$, $B = (-1, 2, -8)$ *and* $C = (4, 5, 0)$.

Problem 33. *Find an equation for the plane that passes through the following points:* $(3, -1, 4), (0, 0, 6)$, *and* $(6, 7, -1)$.

Problem 34. *What is the area of the parallelogram spanned by the vectors* $(3, 1)$ *and* $(2, -2)$*? What is the area of the parallelogram spanned by the vectors* $(1, 1, 1)$ *and* $(-1, 1, 1)$*?*

Problem 35. *What is the area of the parallelogram spanned by the vectors* $\mathbf{u} = 4\mathbf{i} + \mathbf{j} + \mathbf{k}$ *and* $\mathbf{v} = -\mathbf{j}$*?*

Problem 36. *What is the volume of the parallelipiped spanned by the vectors* $\mathbf{u} = (4, 1, 1)$, $\mathbf{v} = (0, 1, 0)$ *and* $\mathbf{w} = (0, 0, 1)$*?*

Problem 37. *What is the volume of the parallelipiped spanned by the vectors* $\mathbf{u} = (0, 0, 1)$, $\mathbf{v} = (1, 1, 2)$ *and* $\mathbf{w} = (1, 1, 0)$*? What is* $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$*?*

Problem 38. *What is the volume of the parallelepiped with sides* $\mathbf{i}$, $4\mathbf{j} - \mathbf{k}$, *and* $8\mathbf{i} + 3\mathbf{j} - \mathbf{k}$*?*

Problem 39. *Consider the parametric equation of the plane* $\pi(t, s) = (1 + t + s, -s, 2 - t + s)$.
(1) *Give two different points in the plane.*
(2) *Find two nonparallel vectors that are parallel to the plane.*
(3) *Find a normal vector to the plane.*
(4) *Write the general equation of the plane, i.e. an equation of the form* $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$.

Problem 40. *Consider the general equation of the plane* $x + y - z = 1$.
(1) *Find a normal vector to the plane.*
(2) *Find a point in the plane.*

(3) *Find two nonparallel vectors that are parallel to the plane.*
(4) *Write a parametric equation of this plane.*

Problem 41. *In 3-dimensional space, two planes that are not parallel intersect in a line. Because given any line it is always possible to find two nonparallel planes that contain it, an alternative way of describing a line is by giving the equations of two planes. For instance, one can talk about the line given by:*

$$\begin{cases} x + y - 2z = 6 \\ x - y = 0 \end{cases}$$

(1) *Find a point in this line.*
(2) *Find the direction vector of this line. (***Hint:** *since the line is contained in both planes, it's direction vector is parallel to both, hence it is perpendicular to the normal vectors of both planes).*
(3) *Find a parametric equation of this line.*
(4) *What is the intersection of this line with the plane $z = -1$? (Notice that you can either use the parametric equation from part c), or intersect the three planes).*